Magnetic Force

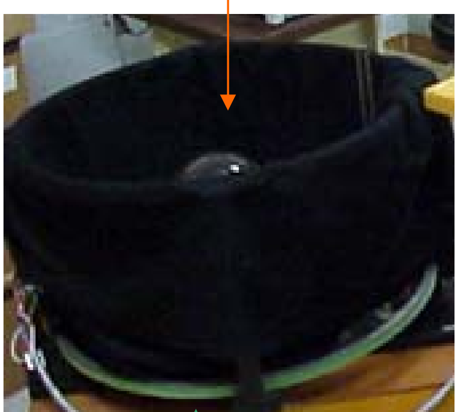
Introduction: The *electron* is a true elementary particle. Even though protons and neutrons, for example, appear to be composed of quarks, there is no indication that electrons are composed of anything else. Their fundamental nature and the fact that electrons are responsible for the operation of all electronic equipment make them important and suggest that we study them if we can. Unfortunately, electrons are so small that we can't observe them directly. Fortunately, they interact with matter so strongly that when they are moving they often leave a "trail." In today's experiment, we will create a beam of electrons and study it via its "trail." That, in itself, should be interesting since a beam of electrons striking a material is what creates the image on the screen in the picture tubes (CRTs) of non-flat screen TVs and computers. We will see that a uniform *magnetic field*, **B**, causes a beam of electrons to travel in a circle if the velocity of the electrons is perpendicular to the magnetic field. In addition, we will try to verify that the magnetic force is given by -*e***v**x**B**, where -*e* is the charge on the electron and **v** is its velocity.

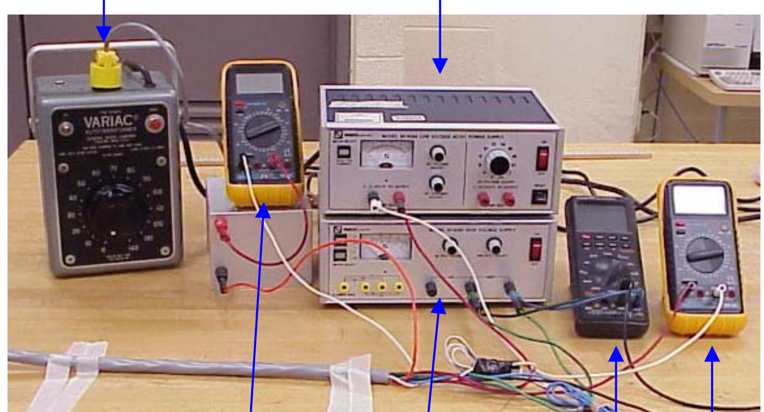
Equipment Pictures of the equipment are shown below.

Glass Tube

Variac (controls the filament current)

Low Voltage Power Supply (controls the current in the coils that creates the magnetic field)





Helmholtz Coils (The upper set of coils is not visible because of the black cloth.)

M1

High Voltage Power Supply (for linearly accelerating electrons)

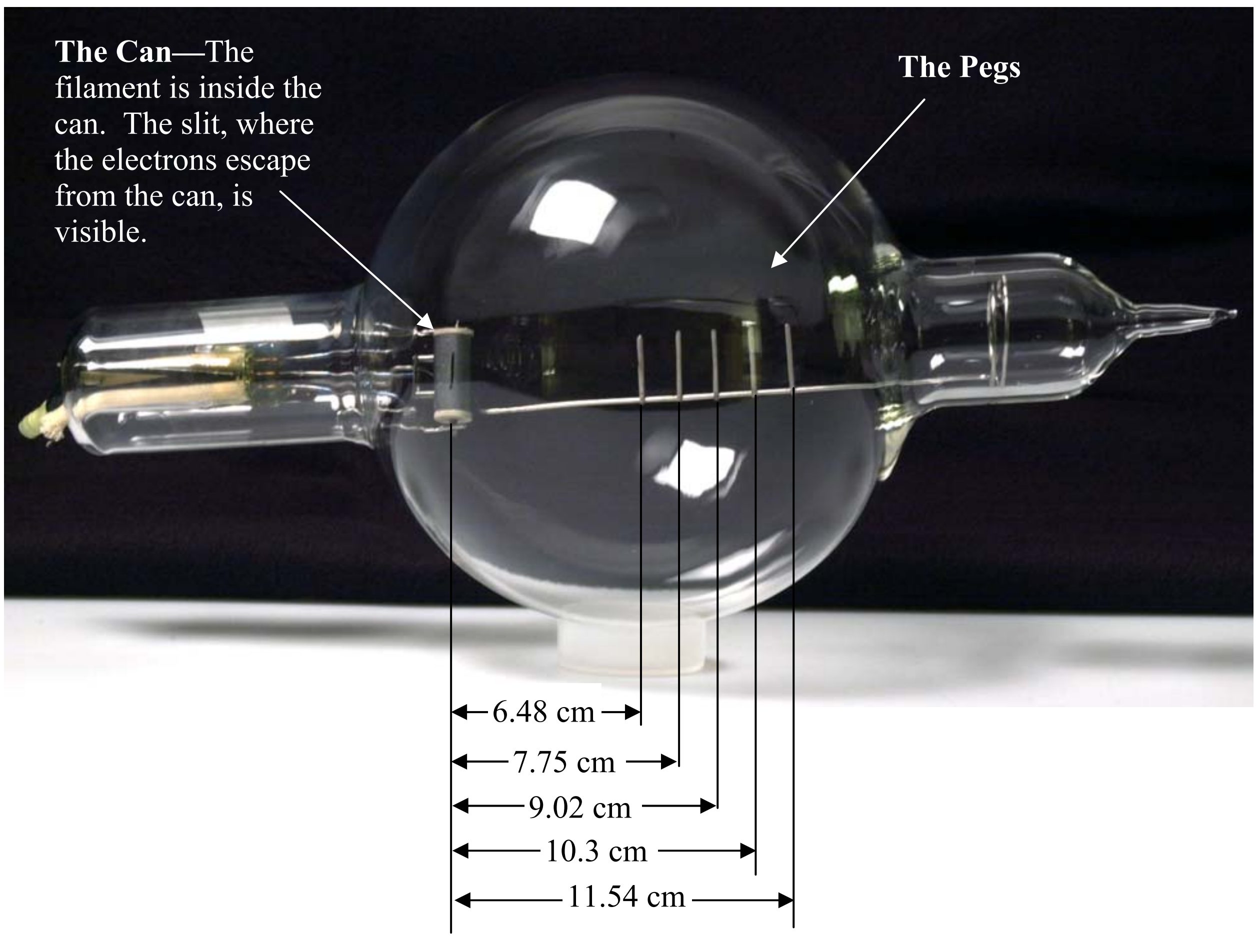
M2

M3

The glass tube, alone, is shown on the next page. The tube is hollow and has mercury and electrical parts inside. The internal structure of the tube is shown schematically on the next page. For this experiment, the following four processes take place inside the glass tube.

1. Electrons are made available for the experiment. Inside the tube is a small, metal can (cylinder about 1.5 cm in diameter and 3.5 cm long with a top and bottom) that contains a *filament*. (We can't see the filament because it is inside the can.) A voltage will be applied to the filament by a *Variac* so that there is current in the filament and it glows much like the filament in a light bulb. In fact, when the filament is hot, we can see the light that it produces because the can has a slit in the side. The important feature for this experiment is that when a filament is at high temperature, many electrons have enough energy to escape from it. (Some electrons just “boil off” the filament.)

The Glass Tube and its Internal Structure



2. *The electrons are accelerated linearly inside the can.* A *High Voltage Power Supply* is connected between the filament and the can so that the can is positive relative to the filament. Consequently, the electrons that boil off the filament will be accelerated toward the can. Most of the electrons just crash into the can. However, those that encounter the slit will escape the can traveling with a velocity that we know how to calculate.

3. *The electron beam is made observable.* If the tube were at atmospheric pressure, the electrons would not travel far because of collisions with the air molecules. Consequently, most of the air has been removed. However, the path of the electrons would still not be observable. To make the trail of the electrons observable, low-pressure mercury vapor exists inside the tube. Drops of mercury are visible inside the tube at the bottom. The mercury vapor enables us to see the path of the electrons because when an electron collides with a mercury atom, the atom emits

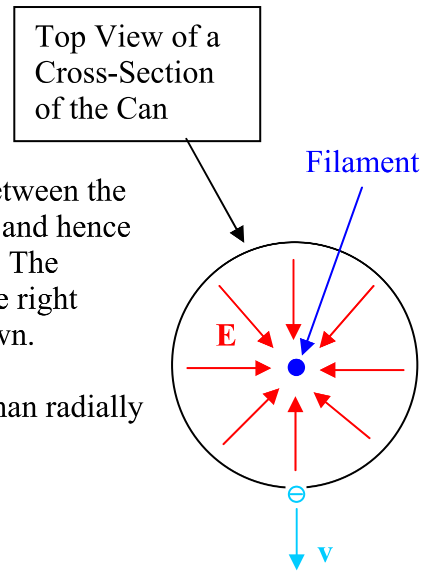
bluish light.

4. *The electron beam is bent into a circle outside the can*. A *magnetic field* causes the electrons to travel in a circle outside the can. The magnetic field is produced by an electric current in a large set of coils *outside the tube*. The coils have a special configuration. There are two sets of circular coils of radius *R* that are separated by the distance *R*. Coils in this configuration are known as *Helmholtz coils*. The special characteristic of Helmholtz coils is that they produce a relatively uniform magnetic field near their middle. Our electron beam will be located near the middle of the coils.

Some jargon: The filament is sometimes known as a *cathode* and the electron beam is sometimes known as a *cathode ray*. These terms should be familiar since we have all heard of a *cathode ray tube or CRT*. Just to confuse us, however, the can *(cylinder)* is sometimes known as the *plate*. We will avoid that terminology.

Theory

1. (Linear) Motion of the Electrons Inside the Can



Since there is a positive potential difference (voltage), V_{cf}, between the filament and the can (cylinder) there is an electric field there and hence an electric force on the electrons that "boil off" the filament. The electric field is radially inward as shown in the drawing at the right where a cross section of the can with slit and filament is shown.

R1: Explain why the electric field is radially inward rather than radially outward and explain why the electric field is not constant.

Since the electric field (and hence electric force) is not constant, we cannot use the kinematic equations for constant acceleration to calculate the speed of the electrons. However, because the electric field is conservative, it is relatively easy to use energy concepts. As the electrons move from the filament to the can, they lose electric potential energy, U_{cf}, given by

$$U_{cf} = -eV_{cf} \qquad (1)$$

$-e$ is the charge on the electron i.e. $q = -e = -1.6\text{x}10^{-19}$ C. As usual, the loss of potential energy is accompanied by an increase in kinetic energy so that

$$\frac{1}{2}mv^2 - eV_{cf} = 0 \qquad (2)$$

m is the mass of the electron, $m = 9.1\text{x}10^{-31}$ kg. Eq. (2) enables us to calculate the speed of the electrons exiting the can since it follows that

$$v = \sqrt{\frac{2eV_{\mathrm{cf}}}{m}} \qquad (3)$$

Hopefully, it is clear that the velocity of the electrons exiting the can (at the bottom of the can) is in the direction shown in the diagram.

2. (Circular) Motion of the Electrons Outside the Can

When the electrons exit the can, they encounter a magnetic field that causes them to move in a circle of radius r as shown in the next diagram. (We assume that there is no magnetic field inside the can.) Since we can predict the speed of the electrons (eq. (3)) and since, in our experiment, r is known, the acceleration and hence the required force can be calculated. Specifically, for motion in a circle of radius r, the acceleration is given by

$$a = \frac{v^2}{r} \qquad (4)$$

Thus, the magnitude of the force required to cause the circular motion is given by

$$F_{\mathrm{req}} = \frac{mv^2}{r} \qquad (5)$$

Beware: Little r is the radius of the orbit of the electrons and big R is the radius (and separation) of the Helmholtz coils.

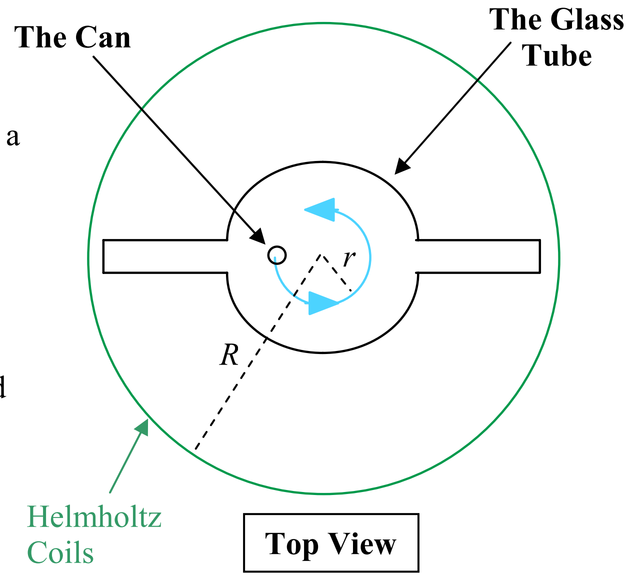
3. The Magnetic Force

Again, the force that causes the circular motion is a magnetic force. It should be apparent that the magnetic force on the electrons is given by

$$\mathbf{F}_{\mathrm{mag}} = -e\mathbf{v}\mathrm{x}\mathbf{B} \qquad (6)$$

Since the magnetic field is perpendicular to the velocity, the magnitude of the magnetic force, and hence the force required to cause the electrons to travel in a circle is given by

$$F_{\mathrm{mag}} = F_{\mathrm{req}} = evB \qquad (7)$$



R2: Referring to the previous sketch, what is the direction of the magnetic field that will cause the electrons to travel in the circle shown?

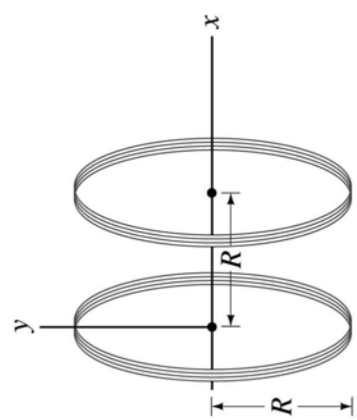
4. The Magnetic Field

As has been mentioned previously, the magnetic field in our experiment will be produced by Helmholtz coils. A top view of the Helmholtz coils is shown in the previous sketch.

R3: Referring to the previous sketch, what is the direction of the current in the Helmholtz coils that will produce the magnetic field that will cause the electrons to travel in the circle shown?

In order to calculate the magnetic field, consider the next figure which is a sketch of Helmholtz coils. The figure is taken from problem 58 on page 733 of the textbook and is rotated 90° to match our experiment. Comparing the figure and Example 28-10 on page 720 of the textbook, it may be apparent that if there is a current I in the coils, the magnetic field between the coils at a distance x from one set of N coils and a distance R - x from the other set of N coils is given by

$$B = \frac{\mu_o INR^2}{2}\left[\frac{1}{\left(R^2 + x^2\right)^{3/2}} + \frac{1}{\left(2R^2 + x^2 - 2Rx\right)^{3/2}}\right] \qquad (8)$$



Finally, since the electron beam is approximately halfway between the coils, $x \approx R/2$. Plugging this into eq. (8), it follows that the magnetic field in the vicinity of the electron beam is

$$B \approx \frac{8N\mu_o I}{5\sqrt{5}R} \qquad (9)$$

***N*=72 for our experiment**. Again, the reason that this configuration is used is that the magnetic field is relatively uniform near the middle i.e. the magnetic field doesn't change much along the axis at the middle. However, our electron beam is not along the axis. In the best case, it circles around the axis. Calculation of the off-axis magnetic field is beyond the scope of this course. Fortunately, those calculations show that the radial uniformly is even better than the axial uniformity. Since they show that the magnetic field is constant to about 0.07 % inside a sphere of radius 0.2 R centered at the middle of the Helmholtz coils, we will take the magnetic field produced by the coils to be constant for our electrons and given by eq. (9).

The magnetic field is also complicated by the presence of the earth's magnetic field, B_E. Toward the end of the laboratory, we will estimate the correction for the earth's magnetic field.

Note: The equipment used in this lab is traditionally used to measure the *charge to mass ratio* for the electron. That follows because eqs. (1), (5) and (7) lead to

$$\frac{e}{m} = \frac{2V_{\mathrm{cf}}}{B^2 r^2} \qquad (10)$$

However, if we just plug into eq. (10) to calculate e/m, we will miss some interesting physics. Consequently, we will do things a different way.

The Experiment ***(Please follow the instructions carefully. Also, please do not change any of the wiring without consulting the technical staff. The equipment that we are about to use is expensive.)***

1. Measure the radius of the Helmholtz coils and record the value in the space provided.

R4: R = ________ ± ________ m

2. Describe (manufacturer, etc.) the meter responsible for each of the following measurements.

R5: Meter M1 (Filament Current) ___________________________________

R6: Meter M2 (Electron Accelerating Voltage, V_{cf}) ___________________________________

R7: Meter M3 (Helmholtz Coil Current, I) ___________________________________

3. Turn on the three digital multimeters. They should be set as follows. M1 should be at 10 on the A~ scale, M2 should be on the $\overline{\overline{\dot{V}}}$/LOGIC scale and M3 should be at 10 on the A $\overline{\cdots}$ scale.)

4. Turn on the High Voltage Power Supply and increase the voltage to about 40 V. Read the value of the voltage on M2 and record the measured value as V_{cf} above the first table (next page) in the space provided.

5. Switch the meter on the front of the High Voltage Power Supply to "milliammeter." The value of this current is proportional to the rate that electrons leave the filament i.e. the beam current. The meter should read zero at the present time since the filament is not hot.

6. ***Before proceeding, be sure that the Variac is off and that the dial is set to zero. (Turn the dial all the way counterclockwise.)***

- ***Assign someone in the group to watch M1 and be sure that it never exceeds 4.5A. Higher currents will burn out the filament and new tubes cost about $1500.***

- ***Assign someone in the group to watch the milliammeter on the front of the High Voltage Power Supply and be sure that it never exceeds 8mA. Higher beam currents will also destroy the tube.***

The filament will now be heated (made to glow) so that the electrons can boil off and consequently be accelerated to create an electron beam. Turn on the Variac and gradually increase the voltage/current until the filament glows and the electron beam is visible.

7. Next, we will check that the Helmholtz coil circuit is working i.e. that we can create a magnetic field. Turn on the Low Voltage Power Supply. Increase the DC current (in the coils) until the electron beam travels in a circle. (Note: It may be necessary to turn up the DC voltage

before increasing the current.) If you are unable to "bend the beam," consult your instructor.

R8: Is the magnetic field created by the Helmholtz coils "up" (toward the sky) or "down" (toward the ground)?

Our first experiment will be to try to estimate the component of the earth's magnetic field, B_E, perpendicular to the coils. We will refer to this as the *perpendicular component of the earth's magnetic field.*

8. Reduce the current in the Helmholtz coils to zero and, from above, look carefully at the beam. It should be slightly curved even though there is no applied magnetic field since the current in the Helmholtz coils is zero. The slight curvature is caused by the perpendicular component of the earth's magnetic field. (If the electron beam reflects nicely from the glass wall, it might be easier to observe the reflected beam since it has a longer path length and thus the curvature is more visible.) The curvature can be cancelled if the Helmholtz coils create a magnetic field of equal magnitude and opposite direction to the perpendicular component of the earth's magnetic field. Adjust the current until the beam straightens. Record the value of the current, I_E, in the space provided. (Yes, this is a small effect and we are only estimating the value.)

R9: I_E = ________ ± ________ A. R10: B_E = ________ ± ________ μT.

9. Use eq. (9) to calculate the magnitude of perpendicular component of the earth's magnetic field at the tube and record the value above in the space provided.

R11: Is the perpendicular component of the earth's magnetic field "up" (toward the sky) or "down" (toward the ground)? (There are horizontal components of the earth's magnetic field but they won't affect our experiment.)

10. Finally, we will bend the electron beam into circles of known radii. Increase the current in the coils until the electron beam strikes the outside of the peg farthest from the can.

R12: Record the current reading from M3 in the second column of the table below.

11. Continue increasing the current so that the beam strikes the outside of each of the pegs. Record each value of the current in the appropriate blank in the table.

R13: V_{cf} = ________ ± ________ V R14: v_{40V} = ________ ± ________ m/s

r (m)	I (A)	B-B_E (μT)	a (m/s^2)	F_{req} (N)	F_{mag} (N)
0.0577 ±					
0.0515 ±					
0.0451 ±					
0.0388 ±					
0.0324 ±					

12. Repeat the experiment for a high voltage of about 80 V. R15: Record the value of V_{cf} in the space provided and the results of your measurements of the current in the next table.

R16: V_{cf} = ________ ± ________ V R17: υ_{80V} = ________ ± ________ m/s

r (m)	I (A)	B-B_E (μT)	a (m/s^2)	F_{req} (N)	F_{mag} (N)
0.0577 ±					
0.0515 ±					
0.0451 ±					
0.0388 ±					
0.0324 ±					

13. Carefully turn off the equipment in the reverse order of the experiment i.e. first turn off the equipment that was turned on last, etc.

Calculations

It is more convenient to carry out the calculations using a spreadsheet. Should you choose to use a spreadsheet, either attach a printout to your report or give your instructor an electronic copy and don't bother filling out the tables. In either case, be sure to answer the questions associated with many of the required calculations.

1. Use eq. (3) to calculate the speed of the electrons for the two accelerating voltages.

R18: Show a sample calculation and record the values of υ_{40V} and υ_{80V} in the space provided.

R19: Is the speed "high" or "low?" Discuss.

2. Use eq. (4) to calculate the magnitude of the acceleration of the electrons.
R20: Show a sample calculation and record the value for each of the situations in the tables.

R21: Are these accelerations "high" or "low?" Discuss.

3. Use eq. (5) to calculate the magnitude of the force required to make the electrons travel in each of the observed paths. R22: Record the values in the tables.

4. Use eq. (9) to calculate the magnitude of the magnetic field due to the Helmholtz coils, B, for each of the situations. R23: Show a calculation.

5. Assuming that your experiment is set up in the usual manner, subtract the value of the earth's magnetic field, B_E from the magnitude of the magnetic field due to the Helmholtz coils, B.

R24: Record the value of the magnitude of the total magnetic field, B-B_E, in the tables.

R25: Why was B_E subtracted from B rather than added to it?

6. Use eq. (7) to calculate the magnitude of the magnetic force. (Be sure to use B - B_E rather than B in eq. (7).) R26: Record the values in the tables.

R27: Does the magnitude of the magnetic force obey the law (N2L) i.e. Is the magnitude of the magnetic force equal to the mass times the acceleration? Discuss.

R28: What is the direction of the magnetic field at the center of the coils due to the circulating electrons? (No we have not considered this magnetic field previously since it is irrelevant for this experiment.)

R29: How does the direction of the magnetic field produced by the electrons compare with the direction of the magnetic field produced by the Helmholtz coils?